First Annual Ohio Math Contest
Exam III – 10th & 11th Grade

Name _____________________  Grade ___________

You will have 1 hour to complete this exam. Calculators are not allowed. Sorry, but the proctors are not allowed to answer any questions. Good luck!

Part I. If you get confused, just take your best guess.

1) The positive integers 30, 72, and N have the property that any two product of them is divisible by the third. What is the smallest possible value of N?  Answer ______________

2) In how many ways can the numbers (1, 2, 3, 4, 5, 6) be ordered such that no two consecutive terms have a sum which is divisible by 2 or 3?  Answer ______________

3) If the 5 digit number 5DDDD is divisible by 6, then find the digit D.  Answer ______________

4) Young’s digital clock read 7:15am when she left for school. When she returned home 7 hours 15 minutes later, the clock read 5:55am because the power had gone off during the day. If the clock automatically reset to 12:00am after the power was restored, at what time that morning did the power return?  Answer ______________

5) What is the remainder when 19^{1999} is divided by 25?  Answer ______________

6) Solve $2x^{1/3} – 3x^{1/6} + 1 = 0$  Answer ______________

7) Factor $4x^2 + 20x +13 = 0$  Answer ______________

8) Solve the following equation. Give the exact value of the solution.  Answer ______________

$$\cot x \cos^2 x = 2 \cot x$$

9) There is a unique positive integer that has less than 11 digits, ends in a 6, and if this 6 is removed and put at the front of the number, then the resulting number is 4 times the original number. How many digits does this number have?  Answer ______________

10) If (6, 9) and (10, 3) are the coordinates of two opposite vertices of a square, what are the coordinates of the other two vertices?  Answer ______________

11) The following inequalities hold for all positive integers N. $\sqrt{n+1} - \sqrt{n} < 1 / \sqrt{4n+1} < \sqrt{n} - \sqrt{n-1}$  What is the greatest integer which is less than $\sum_{n=1}^{24} (4n+1)^{-1/2}$?  Answer ______________

Part II. Support your answer by giving all your work. Just your guess only does not get full credits.

1) A function $f(n)$ defined for all positive integers has the property that $f(m) + f(n) = f(mn)$ for all positive integers m and n. If $f(2) = 7$ and $f(3) =10$, then calculate $f(12)$. Answer ______________

2) There are 6 gallons of pure alcohol in container A and 6 gallons of pure water in container B. An empty bottle is filled with alcohol from container A and then emptied into B. After stirring, the bottle is filled with this mixture from B and emptied into A. The ratio of alcohol to water in container A is now 4 :1. Assuming there were no spills, what is the size of the bottle in gallons?  Answer ______________

3) The squares in the diagram have side length 1. What is the area of the slanted rectangle?  Answer ______________

4) Congruent circles A and B intersect such that $\overline{AB}$ is a radius of each circle. If $\overline{AB} = 6$cm, what is the number of square centimeters in the shaded region? Use 3.14 as an approximation for pi, and express your answer as a decimal to the nearest tenth.  Answer ______________

The end! Congratulations!